### Summary

The foregoing analysis shows that the Clohessy-Wiltshire and Duke matrices are equivalent. They stem, in fact, from the same linearizing approximations. The principal assumptions are 1) the distance between orbiting body and reference satellite is very small compared to the distance to the center of the force field; and 2) the variation of the gravity field is linear in the vertical direction over the region of interest.

Examination of the transformation equations reveals an interesting lack of symmetry, i.e., the horizontal velocity transformation is cross-coupled with the vertical position. Yet no coupling term appears for the vertical velocity. The coupling is caused by the coordinate system rotation. Since the Clohessy-Wiltshire system rotates with respect to inertial space, the transformation between it and a fixed inertial system contains symmetrical cross-coupling terms for each velocity axis. The lack of symmetry in the transformation equations under discussion may be interpreted to mean that the Duke system appears to be rotating when vertical velocities are considered but appears to be fixed for horizontal velocities. Hence the correct translation of velocities for this system to those in an inertial coordinate system requires considerable care.

### References


[1] Clohessy, W. H. and Wiltshire, R. S., "Terminal guidance system for satellite rendezvous," IAS Paper 59-93 (June 1959).

[2] Duke, W. M., Goldberg, E. A., and Pfeffer, I., "Error analysis considerations for a satellite rendezvous," ARS J. **31**, 550–513 (1961).

[3] Wisneski, M. L., "Error matrix for a flight on a circular orbit," ARS J. **32**, 1416–1418 (1962).


# Particle Damping of a Plane Acoustic Wave in Solid Propellant Combustion Gases


David W. Blair*

*Norwegian Defence Research Establishment, Kjeller, Norway*


### Nomenclature

$a$ = particle radius, cm
$c$ = velocity of sound in the gas, cm/sec
$c_v$ = specific heat of the gas at constant volume, cal/g-°K
$E$ = energy flux in the plane wave, erg/cm²-sec
$f$ = acoustic frequency, cps
$H''$ = defined by Eq. (2), dimensionless
$k$ = $\omega/c$, cm$^{-1}$
$L$ = decay length for all three mechanisms combined, cm
$L_b$ = decay length for bulk damping of the pure gas phase, cm
$L_p$ = decay length for particle damping, cm
$L_w$ = decay length for pure gas phase wall damping, cm
$N$ = particle number density in combustion products, cm$^{-3}$
$P_0$ = equilibrium pressure, dynes/cm²
$R$ = tube radius, cm
$V$ = volume of condensed products per unit volume of total combustion products
$x$ = distance, cm
$\beta$ = $(\omega/2\nu)^{1/2}$, cm$^{-1}$
$\gamma$ = ratio of specific heats
$\lambda$ = gas phase thermal conductivity, cal/sec-cm-°K
$\mu$ = dynamic viscosity, poise
$\nu$ = kinematic viscosity, stokes
$\rho_0$ = gas phase density, g/cm³
$\rho_1$ = solid particle density, g/cm³
$\sigma$ = acoustic damping constant, cm$^{-1}$
$\omega$ = circular frequency, rad/sec


Received March 1, 1963; revision received July 1, 1963. The author gratefully acknowledges the support of the Royal Norwegian Council for Scientific and Industrial Research and the Division for Explosives, Norwegian Defence Research Establishment, during the course of this work.



* Postdoctoral Fellow of the Royal Norwegian Council for Scientific and Industrial Research; presently Associate Professor of Mechanical Engineering, Polytechnic Institute of Brooklyn. Member AIAA.


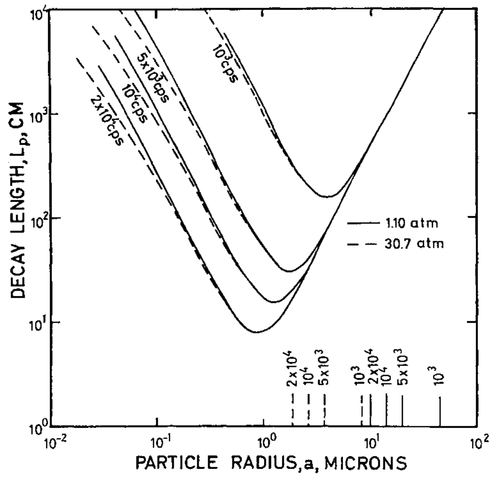


**Fig. 1 Decay length for particle damping as a function of particle radius. For the indicated frequencies, the short vertical lines give the largest particle radii for which the calculation is valid. The calculation was restricted to $\beta a <$ 10–1.**

It is a matter of general knowledge that acoustic combustion instability in solid propellant rocket motors can often be cured by adding certain substances to the solid propellant itself. These substances may or may not participate in the chemical reactions of the combustion process, but they always produce solid or liquid particles in the combustion gases. Additives such as Al and Mg powders, which participate in the combustion reactions, appear to be more effective in suppressing acoustic instability than do inert additives.

The mechanism by which these additives suppress instability is not known. They may act by decreasing the effectiveness of the acoustic amplifiers in the motor or by increasing the acoustic losses. It is the purpose of this note to present calculations that investigate the latter possibility. The viscous damping that is caused by the presence of solid or liquid particles in the combustion gases is calculated and compared with that of the pure gas phase without particles. The particular case of a propellant that contains 10% aluminium by weight is considered. A plane acoustic wave is assumed, and the effect of particle size on the decay length of the wave is calculated for frequencies from $10^3$ to $2 \times 10^4$ cps and for pressures of 1 and 28 atm. The results are compared with the decay length for thermal and viscous damping in the bulk of the pure gas phase, and with the decay length for thermal and viscous wall damping of the pure gas phase when the wave is traveling axially along a cylindrical tube of 5-cm radius.

Previous work concerning the effect of particle damping on a particular mode of oscillation in a solid propellant rocket motor may be found in Ref. 1.

### Analysis

Reference 2 gives the results of a theory of the energy absorbed by viscous damping from a plane acoustic wave as it passes over a spherical particle that is free to move.

For the case where the plane wave is propagated through a medium that contains many such particles, all of radius $a$, and where the particles are not so closely spaced that they interact with one another, the rate of dissipation of acoustic

**Table 1 Gas phase decay lengths for both bulk and wall damping in a tube of 5-cm radius**

| | $P = 1.1$ atm | | $P = 30.7$ atm | |
|---|---|---|---|---|
| $f$, cps | $L_b$, cm | $L_w$, cm | $L_b$, cm | $L_w$, cm |
| $10^3$ | $3.26 \times 10^6$ | $1.42 \times 10^3$ | $9.09 \times 10^7$ | $7.50 \times 10^3$ |
| $5 \times 10^3$ | $1.30 \times 10^5$ | $6.33 \times 10^2$ | $3.64 \times 10^6$ | $3.35 \times 10^3$ |
| $10^4$ | $3.26 \times 10^4$ | $4.47 \times 10^2$ | $9.09 \times 10^5$ | $2.36 \times 10^3$ |
| $2 \times 10^4$ | $8.12 \times 10^3$ | $3.17 \times 10^2$ | $2.26 \times 10^5$ | $1.68 \times 10^3$ |

energy through viscous interaction with the particles is given by

$$dE/E = -3kVH''dx = -4kaNH''\pi a^2 dx \qquad (1)$$

with greater than the optimum particle size. Thus, the analysis indicates the desirability of adding the powders in the condition of the finest granulation that is obtainable. As would be expected, the damping increases as the quantity of additive increases [Eq. (3)].

The analysis takes no account of the chemical reactivity of the additives. This might profoundly alter their damping characteristics. The particle size of the products might, for example, be considerably different from that of the additive itself. Also, the temperature nonequilibrium between the particles and the gas and the jetting and spinning of the burning particles will probably change their damping properties.

$$H'' = \frac{\{12\}\{1 + \beta a\}\{[\beta a]^2 + [\delta/(1-\delta)][(\beta a)^2 + \frac{3}{2}\beta a]\}}{16[\beta a]^4 + 48[\delta/(1-\delta)][(\beta a)^4 + \frac{3}{2}(\beta a)^3] + 81[\delta/(1-\delta)]^2[\frac{4}{9}(\beta a)^4 + \frac{4}{3}(\beta a)^3 + 2(\beta a)^2 + 2(\beta a) + 1]} \qquad (2)$$

Equation (2), which is obtained from the results of Refs. 2–4, holds for $\beta a \ll 1$. When $\delta \ll \beta a$ and higher-order terms are dropped, Eq. (2) agrees with the result of Ref. 8. In the foregoing equations, $k = \omega/c$, $\beta = (\omega/2\nu)^{1/2}$, $\delta = \rho_0/\rho_1$, and $V$ is the volume of solid particles per unit volume of total mixture, medium, and solid particles.

The decay length $L_p$ over which the acoustic energy flux is reduced to $1/e$ of its original value is given by

$$L_p = 1/3kVH'' \qquad (3)$$

The viscous and thermal damping by the pure gas phase, without particles, consists of damping both in the bulk of the gas and at the walls of the tube. For the case of a plane wave traveling axially along a tube of radius $R$, the two damping constants are given by Refs. 5–7:

$$\sigma_b = \left[\frac{2\pi^2\mu}{\gamma c}\right]\left[\frac{4}{3} + \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{\lambda}{\mu c_v}\right)\right]\left[\frac{f^2}{P_0}\right] \qquad (4)$$

$$\sigma_w = \left[\frac{(\mu\pi)^{1/2}}{R}\right]\left[\left(\frac{1}{\gamma}\right)^{1/2} + \left(\frac{\lambda}{\mu c_v}\right)^{1/2}\left(\frac{\gamma - 1}{\gamma}\right)\right]\left[\frac{f}{P_0}\right]^{1/2} \qquad (5)$$

and the decay length is given by

$$L_b = 1/2\sigma_b \qquad L_w = 1/2\sigma_w \qquad (6)$$

In the case where all three types of damping are present and acting independently, the combined damping constant is given by

$$\frac{1}{L} = \frac{1}{L_p} + \frac{1}{L_b} + \frac{1}{L_w} \qquad (7)$$

Equations (2–5) were used to calculate the decay lengths $L_p$, $L_b$, $L_w$ for a hypothetical propellant that contained 10% Al by weight. It was assumed that all of the aluminium was converted to liquid $Al_2O_3$.

The combustion gases were assumed to have the properties of CO at 3000°K. Thus, the physical constants had the following values: $c_v = 0.227$ cal/g-°K, $c = 1.12 \times 10^5$ cm/sec, $\lambda = 2.6 \times 10^{-4}$ cal/sec-cm-°K, $\gamma = 1.4$, $\mu = 7.75 \times 10^{-4}$ poise, $\rho_1 = 3.5$ g/cm³, $\rho_0 = 1.14 \times 10^{-4}$ g/cm³ per atmosphere pressure, $V = 0.233\ (\rho_0/\rho_1) = 0.0665\ \rho_0$.

The calculation was made for pressures of 1.1 and 30.7 atm. The results for $L_p$ are shown in Fig. 1, whereas those for $L_w$ and $L_b$ (for $R = 5$ cm), are shown in Table 1.

It is noteworthy that the particle damping is relatively insensitive to chamber pressure, whereas the wall damping is inversely proportional to the square root of the chamber pressure. Thus, over the range of critical particle sizes where particle damping far exceeds wall damping, $L$ will be insensitive to pressure level.

The maximum particle damping occurs in the size range 1 to 10 $\mu$, which is a range that is available in commercial metal powders. Powders of smaller particle size are difficult to obtain, and most propellant systems probably operate

## Conclusions

The combustion products from metal additives in solid propellants can greatly increase the acoustic damping constant of the combustion gases. The particle sizes for which this effect is most pronounced are in the 1- to 10-$\mu$ range, which is about the minimum size range in which commercial metal powders are available. When the damping is predominately caused by particles in the combustion gases, it is insensitive to chamber pressure.

In Ref. 9, Horton and McGie have presented results of their calculations of the acoustic damping constant for propellent gases that contain 2% of $Al_2O_3$ particles. This analysis is part of a larger analysis of their experimental results, and their viewpoint is somewhat different than that taken in this note.

## References


[1] Bird, J. F., McClure, F. T., and Hart, R. W., "Acoustic instability in the transverse modes of solid propellant rockets," Johns Hopkins Univ., Appl. Phys. Lab. TG 335-8 (June 1961).

[2] Lamb, H., *Hydrodynamics* (Dover Publications, New York, 1945), 6th ed., pp. 655–661.

[3] Lamb, H., Ref. 2, Eq. (44), p. 661.

[4] Lamb, H., Ref. 2, Eq. (32), p. 657.

[5] Parker, J. G., "Effect of several light molecules on the vibrational relaxation time of oxygen," J. Chem. Phys., 1763–1772 (1961).

[6] Rayleigh, *The Theory of Sound* (Dover Publications, New York, 1945), pp. 325–326.

[7] Herzfeld, K. F., *Thermodynamics and Physics of Matter* (Princeton University Press, Princeton, N. J., 1955), Sec. H, p. 660.

[8] Epstein, P. S. and Carhart, R. R., "The absorption of sound in suspensions and emulsions. I. Water fog in air," J. Acoust. Soc. Am. **25**, 553–565 (1953).

[9] Horton, M. D. and McGie, M. R., "Particulate damping of oscillatory combustion," AIAA J. **1**, 1319–1326 (1963).


# Elliptic Elements in Terms of Small Increments of Position and Velocity Components


J. Pieter de Vries*
*General Electric Company, Valley Forge, Pa.*


### Nomenclature

***Elliptic elements***

$a$ = semimajor axis
$L$ = mean longitude, measured from $\xi$ axis


Received March 25, 1963; revision received August 26, 1963.



* Manager, Astrodynamics, Space Sciences Laboratory. Member AIAA.